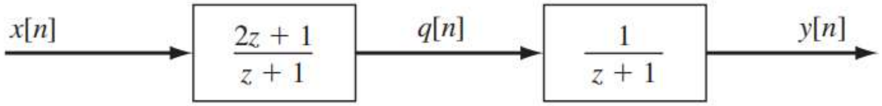
7.36. A linear time-invariant discrete-time system is given by the cascade connection shown in Figure P7.36.

(a) Compute the unit-pulse response of the overall system.
(b) Compute the input/output difference equation of the overall system.
(c) Compute the step response of the overall system.
(e) Compute $y[n]$ when $x[n] = (0.5)^n u[n]$ with $y[-2] = 2$, $q[-2] = 3$.
(f) Verify the results of part (a) and parts (c) to (e) via computer simulation.



注：该解题过程参考教材

FIGURE P7.36

(a) $H(z) = \frac{2z+1}{z+1}\frac{1}{z+1} = \frac{2z+1}{(z+1)^2} = -2\frac{-z}{(z+1)^2} + \frac{1}{(z+1)^2}$

$na^n u[n] \leftrightarrow \frac{az}{(z-a)^2}$

$x[n+1] \leftrightarrow zX(z) - x[0]z$

$$h[n] = -2n(-1)^n u[n] - (n-1)(-1)^{n-1}u[n-1]$$

(b) $H(z) = \frac{2z+}{z+1}\frac{1}{z+1} = \frac{2z+1}{(z+1)^2} = \frac{2z+1}{z^2+2z+1} = \frac{2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} = \frac{Y(z)}{X(z)}$

$$y[n] + 2y[n-1] + y[n-2] = 2x[n-1] + x[n-2]$$

(c) $Y(z) = X(z)H(z) = \frac{2z+1}{z^2+2z+}\frac{z}{z-1} = \frac{c_1}{z-1} + \frac{c_2}{z+1} + \frac{c_3}{(z+1)^2}$

详见课本p374页，7.70

$u[n] \leftrightarrow \frac{z}{z-1}$

$$c_1 = [(z-1)Y(z)]_{z=1} = \frac{3}{4}$$

$$c_3 = [(z+1)^2 Y(z)]_{z=-1} = -\frac{1}{2}$$

$$c_2 = [\frac{d}{dz}(z+1)^2 Y(z)]_{z=-1} = \frac{5}{4}$$

利用y(n+1)=zY(z)

$c_2 n(p_1)^{n-1}u[n] \leftrightarrow \frac{c_2 z}{(z-p_1)^2}$

(c) 答案

$$y[n] = \frac{3}{4}u[n-1] + \frac{5}{4}(-1)^{n-1}u[n-1] - \frac{1}{2}(n-1)(-1)^{n-2}u[n-1]$$

(e)

$$y[n]+2y[n-1]+y[n-2]=2x[n-1]+x[n-2]$$

$x[n-1] \leftrightarrow z^{-1}X(z)+x[-1]$

$x[n-2] \leftrightarrow z^{-2}X(z)+x[-2]+z^{-1}x[-1]$

$x[n]=(0.5)^n u[n]$ with $y[-2]=2, q[-2]=3.$

$$Y(z)+2z^{-1}Y(z)+y[-1]+z^{-2}Y(z)+y[-2]+z^{-1}y[-1]=2z^{-1}X(z)+z^{-2}X(z)$$

$$\frac{Y(z)}{Q(z)}=\frac{1}{z+1}$$

$$y[n+1]+y[n]=q[n]$$

$$y[-1]=q[-2]-y[-2]=1$$

本题因为有初始值，输出不仅仅由X(z)H(z)决定（红色框，零状态），还由初值决定（黄色框，零输入）。

$$X(z)=\frac{z}{z-1/2}$$

$$Y(z)(1+2z^{-1}+z^{-2})+1+2+z^{-1}=(2z^{-1}+z^{-2})X(z)$$

$$Y(z)=\frac{-3-z^{-1}+(2z^{-1}+z^{-2})X(z)}{(1+2z^{-1}+z^{-2})}=\frac{-3z^2-z+(2z+1)X(z)}{(z^2+2z+1)}=\frac{(-3z^2-z)(z-1/2)+(2z+1)z}{(z^2+2z+1)(z-1/2)}$$

$$=\frac{-3z^3+\frac{5}{2}z^2+\frac{3z}{2}}{(z^2+2z+1)(z-1/2)}$$

$$\frac{Y(z)}{z}=\frac{c_1}{z-1/2}+\frac{c_2}{z+1}+\frac{c_3}{(z+1)^2}$$

$$c_1=[(z-1/2)\frac{Y(z)}{z}]_{z=1/2}=\frac{8}{9}$$

$$c_3=[(z+1)^2\frac{Y(z)}{z}]_{z=-1}=\frac{8}{3}$$

计算系数方法，详见课本p374页，7.70

$$c_2=[\frac{d}{dz}(z+1)^2\frac{Y(z)}{z})]_{z=-1}=\left[\frac{-6z+\frac{5}{2}}{(z-1/2)}-\frac{-3z^2+\frac{5}{2}z+\frac{3}{2}}{(z-1/2)^2}\right]_{z=-1}=-\frac{17}{3}+\frac{16}{9}=-\frac{35}{9}$$

$$\frac{Y(z)}{z} = \frac{c_1}{z-1/2} + \frac{c_2}{z+1} + \frac{c_3}{(z+1)^2}$$

$$c_2 n (p_1)^{n-1} u[n] \leftrightarrow \frac{c_2 z}{(z-p_1)^2}$$

（e）答案

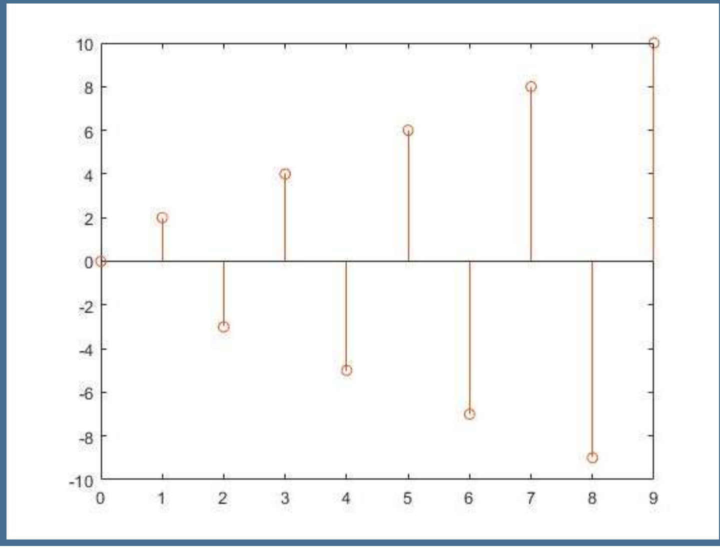
$$y[n] = \frac{8}{9}(\frac{1}{2})^n u[n] - \frac{35}{9}(-1)^n u[n] + \frac{8}{3} n(-1)^{n-1} u[n]$$

(f)

(a)使用impz函数与计算结果对比

因为 $H(z) = \frac{2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$

```
B=[0,2,1];A=[1,2,1];%注意B系数，若用带z的函数，尽量转化为z^-1的式子，再写系数
[h0,n0]=impz(B,A);%计算该系统的单位脉冲响应
for n=n0(1):n0(end)
h1(n+1)=-2*n*(-1)^n*(n>=0)-(n-1)*(-1)^(n-1)*(n>=1);%计算结果
end
figure
%对比结果
stem(n0,h0);
hold on
stem(n0,h1);
hold off
```

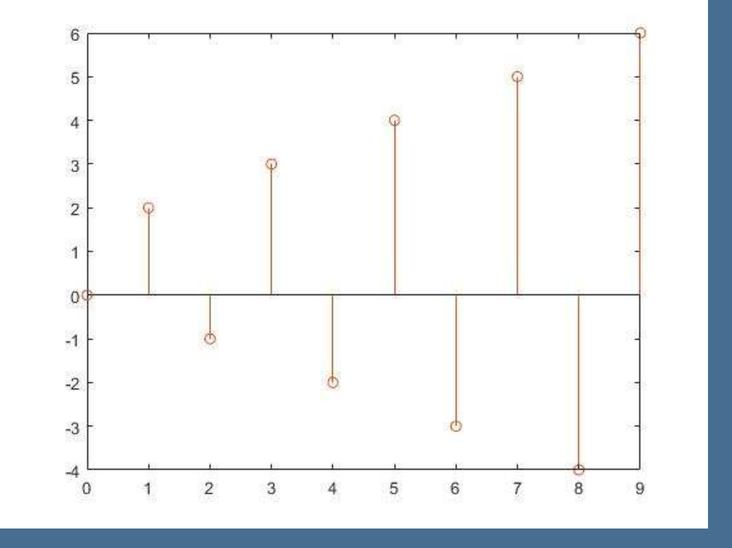


结果一致

(c)使用stepz和计算结果对比

因为 $H(z)=\frac{2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$

```
B=[0,2,1];A=[1,2,1];[s0,n0]=stepz(B,A);%计算阶跃响应
for n=n0(1):n0(end)
s1(n+1)=3/4*(n>=1)+5/4*(−1)^(n−1)*(n>=1)−1/2*(n−1)*(−1)^(n−2)*(n>=1);
end
figure
stem(n0,s0);
hold on
stem(n0,s1);
hold off
```

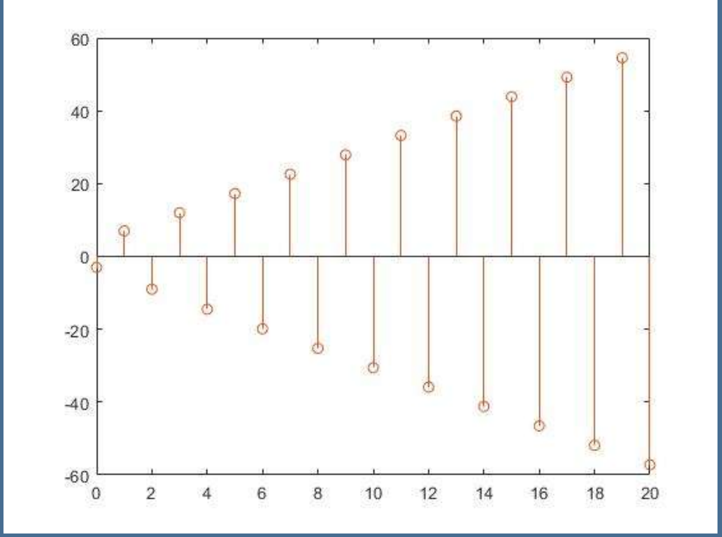


结果一致

(e)使用filter函数和计算结果对比

因为 $H(z)=\frac{2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$ $\quad Y(z)=\frac{-3-z^{-1}+(2z^{-1}+z^{-2})X(z)}{(1+2z^{-1}+z^{-2})}$

```
B=[0,2,1];A=[1,2,1];
n0=0:20;
xn=0.5.^n0;
zi=[-3,-1];%初始状态
yn0=filter(B,A,xn,zi);%使用filter函数计算系统输出结果
for n=0:20
yn1(n+1)=8/9*(1/2)^n*(n>=0)−35/9*(−1)^n*(n>=0)+8/3*n*(−1)^(n−1)*(n>=0);
end
figure
stem(n0,yn0);
hold on
stem(n0,yn1);
hold off
```



结果一致